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Enhancing traffic capacity for scale-free networks by the one-way links

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Abstract

We propose a method to enhance the traffic handling capacity of scale-free networks by enforcing the undirected links to be unidirectional ones, for the global shortest-path routing strategy. By considering three different strategies to determine the directions of unidirectional links, we find that the traffic capacity of networks is considerably improved after applying the link-directed method, especially for the method with non-random direction-determining strategies. Due to the strongly improved network capacity, easy realization on networks and low cost, the method may be useful for modern communication networks.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Perhaps one of the main reasons for a burst of interests in complex networks in the past decade is that many systems in the real world, either naturally evolved or artificially designed, are indeed organized in a networked fashion [1–4]. Rapid advance in network theory greatly enhances our understandings in complex networked topologies and makes applications of network theory in many different fields possible. In particular, features and dynamics of large-scale transportation networks, such as the Internet [5], power grids [6], world wide airports [7] and urban traffic systems [8, 9], have recently attracted a large amount of interest from the physics community due to their importance in our daily life. The ultimate goal of studying these large transportation networks is to control the increasing traffic congestion and improve the efficiency of information transportation. Many recent studies have focused on the efficiency improvement of communication networks which is usually considered from two aspects: modifying the underlying network structure [10–12] or developing better routing

strategies [13–18]. In view of the high cost of changing the underlying structure, developing a better route searching strategy is usually preferable to enhance the network capacity. Packets are suggested to be forwarded using different routing strategies, including the random walk [19, 20], the shortest path [21, 22], the efficient path [17], the nearest-neighbor and next-nearest-neighbor searching strategy [16, 23–25], or the integration of static and dynamic information [15, 24]. The traffic dynamics on networks have been successfully simulated with local or global information to minimize the packet delivery time or maximize the capacity of huge communication networks.

However, it has also been revealed that the traffic dynamics depends strongly on the topology of the underlying networks [26, 27]. Guimerà *et al* proved that homogeneous networks can bear more traffic because of the absence of high-betweenness nodes [10]. This conclusion is also demonstrated by systematic simulations of the traffic on scale-free and homogeneous networks [28]. However, most of realistic networks are scale free [29], which indicates that these networks are prone to the occurrence of the traffic congestion. In this light, some methods to improve, by adding or closing some key links, the overall handling and delivery capacity of scale-free communication networks were proposed [12, 30, 31]. In this paper, we present another alternative method to improve, by limiting some key links to be unidirectional ones, the overall handling and delivery capacity of scale-free communication networks. This link-directed method is also inspired by congestion alleviation in urban traffic systems, in which some highly congested streets are generally limited to one-way ones to reduce the risk of traffic congestion. The strategy is carried out under a global shortest-path routing strategy. It is found that the network capacity is effectively improved.

2. The model

As a paradigmatic example of scale-free networks, we consider the Barabási and Albert (BA) model which has been universally used as a general structure to study various dynamical processes taking place on complex networks [32]. This model introduces two simple mechanisms that are believed to be common in reality: growth and preferential attachment, to construct the SF networks [29]. We follow the algorithm devised in [29]: starting with a small number m_0 of nodes, and then at every time step a new node is added, with m links that are connected to an old node i with k_i links according to the probability $k_i / \sum_j k_j$. After iterating this scheme a sufficient large number of times, we obtain a network composed of N nodes with connectivity distribution $P(k) \propto k^{-3}$ and average connectivity $\langle k \rangle = 2m$. In this paper, the parameters are set to be $m_0 = m = 5$, with network size $N = 5000$.

Several models have been proposed to simulate packet traffic dynamics on complex networks by introducing random of packets at each time step and various routing strategies. We here adopt one widely used before, which is described as follows: at each time step, there are R packets generated in the system, with randomly chosen sources and destinations. It is assumed that all the routers have the same capabilities in delivering and handling information packets, that is, at each time step every node can deliver at most C packets one step toward their destinations according to the global shortest-path routing strategies. When the node cannot transfer all the packets accumulated in its queue, it deals with them following the first-in-first-out rule. For simplicity, we set $C = 1$. The routing strategy adopted in this paper is the global shortest-path routing one, i.e. the packets are forwarded following their shortest path from source to destination. When a packet reaches its destination, the packet will be removed from the system. This model defines the capacity of networks measured by the critical generating rate R_c . At this critical rate, a continuous phase transition from the free flow state to the congested state occurs. In the free-flow state, the numbers of created

and delivered packets are balanced, leading to a steady state, while in the jammed state, the number of accumulated packets increases with time due to the limited delivering capacity or finite queue length of each node [33]. In order to describe the phase transitions of traffic flow in the network accurately, we use the order parameter introduced by Arenas *et al* [27]:

$$\eta(R) = \lim_{t \rightarrow \infty} \frac{C}{R} \frac{\Delta N_p}{\Delta t}, \quad (1)$$

where $\Delta N_p = N_p(t + \Delta t) - N_p(t)$, $\langle \cdot \cdot \cdot \rangle$ indicates the average over time windows of width Δt and $N_p(t)$ represents the number of data packets within the network at time t . For $R < R_c$, $\langle \Delta N_p \rangle = 0$ and $\eta = 0$, indicating that the system is in the free-flow state. However, for $R > R_c$, η increases rapidly from zero, and the system becomes seriously congested [16, 17, 22–25]. Therefore, R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning, and is usually used as a measure of the overall capacity of the system.

3. The link-directed method

The link-directed method first ranks the links according to the value of the product $(k_m \times k_n)$, where k_m and k_n are the links' end-node degrees. Then according to this order from big to small, the links are modified to allow the packets to travel in one direction. Because hub nodes are usually more important and bear more traffic load, the links with bigger values of $(k_m \times k_n)$ are easier to jam. Hence, directing some highly congested links can lead to the redistribution of traffic loads along links, so as to enhance the overall packet handling and delivering ability. As a remark, this paper uses the value of $(k_m \times k_n)$ instead of the links' betweenness centrality, which measures exactly the expected number of packages flowing through the link, because (a) it has been found that the betweenness centrality of links has strong correlation (almost linear) with $(k_m \times k_n)$ [34]; (b) in real weight networks, the weight of links (or traffic load) is proportional to the power of $(k_m \times k_n)$ [35, 36]; and (c) it is easier to rank the links by using the local information, since the calculation of betweenness centrality needs system-wide information.

In order to find a more efficient way to improve the network capacity, we assume that the directions of the unidirectional links can be determined by the links' end-node degrees, according to three different direction-determining (DD) strategies as follows.

- *HTLDD*. The direction of a unidirectional link is fixed from the end-node with high degree to another end-node with low degree. If the degrees of two end-nodes of a unidirectional link are equal with each other, the direction is random.
- *RDD*. The direction of a unidirectional link is fixed randomly.
- *LTHDD*. The direction of a unidirectional link is fixed from the end-node with low degree to another end-node with high degree. If the degrees of two end-nodes of a unidirectional link are equal to each other, the direction is random.

4. The results

According to the link-directed method, we assume that a certain fraction f of links is unidirectional, which is defined as the ratio of the number of unidirectional links L_d to the number of undirected links L_u in initial network, that is

$$f = \frac{L_d}{L_u}. \quad (2)$$

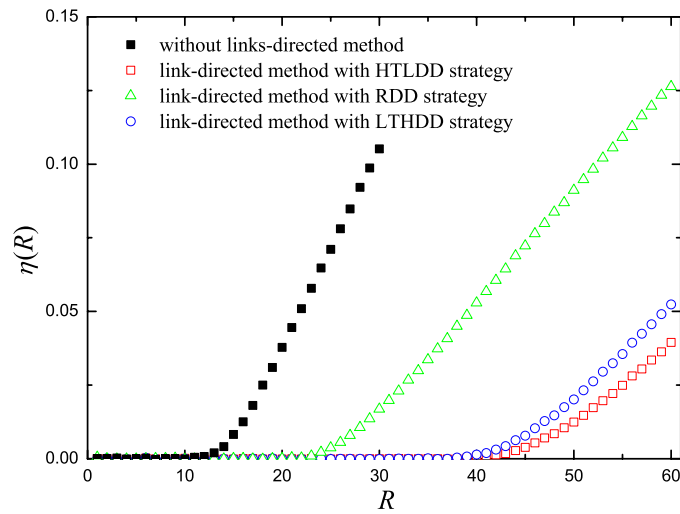


Figure 1. The order parameter η versus R on the BA network for the link-directed methods with different direction-determining strategies. The ratio of the number of directed links to the number of undirected links in the initial network $f = 0.07$. The network parameters are $N = 5000$, $m_0 = m = 5$ and the packet-delivering capacity of the network is $C = 1$. Each data point is obtained by averaging over 200 samples (a set of 20 different realizations of network, and 10 independent experiments for each of them).

Then the directions of these links are given according to three different DD strategies mentioned above. We investigate the changes of some traffic properties such as the order parameter $\eta(R)$, the system's overall capacity R_c , average path length of the packets L_{ave} and the average travelling time $\langle T \rangle$ of the packets.

Figure 1 shows the order parameter η as a function of the generating rate R for the BA network with and without the link-directed method. From this figure, one can find that for each case, when R is less than a specific value R_c , η is zero; it increases suddenly when R is slightly larger than R_c . Moreover, R_c is different for different cases. For the BA scale-free network with $N = 5000$ and $m = 5$, when only 7% links are limited to unidirectional ones, R_c is observably larger than that without a link-directed method, which indicates that the maximal handling and delivering capacity of the system is remarkably enhanced by the link-directed method. In addition, R_c is almost equivalent for the link-directed method with the HTLDD and LTHDD strategies, while significantly larger than that for the link-directed method with RDD strategy. For the initial undirected network, a undirected link can be viewed as two opposite-directed links. Therefore, in the initial network, there exists a strongly positive correlation between the in- and out-degrees of the nodes, i.e. nodes with a large number of outgoing links also have a large number of incoming ones (in fact, here the in- and out-degrees of a node are equal). When applying the link-directed methods with the different DD strategies, the level of correlation between in- and out-degrees will be reduced, especially for the HTLDD and LTHDD strategies. In the directed networks, it seems likely (although this question has not been investigated to our knowledge) that high level of positive correlation between in- and out-degrees are prone to the currency of congestion. Thus, these DD strategies, especially for the non-random DD strategies, that greatly decrease the level of the positive correlation can alleviate traffic congestion.

Generally, different f corresponds to different R_c . To estimate the value of R_c at different f for the link-directed method with three different DD strategies, one can calculate the betweenness centralities of each node, which is defined as [37]

$$g_u = \sum_{s \neq t} \frac{\sigma_{st}(u)}{\sigma_{st}}, \quad (3)$$

where σ_{st} is the number of the shortest paths going from s to t and $\sigma_{st}(u)$ is the number of the shortest paths going from s to t and passing through u . It is well known that for low values of R the system reaches a steady state in which $N_p(t)$ fluctuates around a finite value. As R increases, the system undergoes a continuous phase transition to a congested phase. Below the critical value R_c , there is no accumulation at any node in the network and the number of packets arriving at node v is, on average, $Rg_v/(N(N-1))$. Therefore, a particular node will collapse when $Rg_v/(N(N-1)) > C_v$, where g_v is the betweenness and C_v is the transferring capacity of node v . Considering the transferring capacity of each node is fixed to 1 in this paper and congestion occurs at the node with the largest betweenness, R_c can be estimated as [10, 17, 22]

$$R_c = \frac{N(N-1)}{g_{\max}}, \quad (4)$$

where g_{\max} is the largest betweenness centrality of the network. In figure 2, we report the simulation results for the critical value R_c as a function of f on the BA network, which is in good agreement with the analysis. As one can see, R_c monotonically increases with f for the link-directed method with the HTLDD and LTHDD strategies, while first increases with f and then slowly decreases for the RDD strategy. In comparison with the case without the link-directed method (i.e. $f = 0.0$), the capability of the network in freely handling information is greatly improved, from $R_c = 11$ when $f = 0.0$ to $R_c = 44$ when $f = 0.078$ for the HTLDD strategy, to $R_c = 41$ when $f = 0.078$ for the LTHDD strategy, and to $R_c = 26$ when $f = 0.10$ for the RDD strategy. Apparently, among these DD strategies, the HTLDD one can most effectively enhance the network overall capacity, in which R_c increases about four times. This great improvement is because when applying the link-directed method, the heavy load on central nodes (with the highest connectivity) is strongly redistributed to those nodes with a lower degree. However, as f is even further increased, for the link-directed method with both the HTLDD and LTHDD strategies, R_c vanishes when f is larger than a critical value, since the communication paths among some nodes disappear. For the link-directed method with the RDD strategy, R_c slowly decreases and then vanishes when f is further increased, which indicate that the network is more robust for the RDD strategy, but it cannot effectively enhance the capacity of the network when overmany links are limited to unidirectional ones.

When f is larger than a critical value f_c , the network is fragmented into several disconnected components, which causes the congestion to occur whatever the generating rate R is. So, a practical and important problem: how much can one change the link's directionality to obtain the maximal network capacity, is posed. Likely, this issue can be mapped to the well-known bond percolation problem. However, there is indeed underlying difference between them which leads to a great difficulty to provide an analytical solution by the percolation theory. Moreover, f_c is sensitive to networked structures, and many statistical properties, such as the average degree, the degree distribution and the degree-degree correlation, play an important role on it. Also f_c is closely related to the DD strategies. Therefore, numerical simulation cannot give a generalization of the critical fraction for various networks. The BA network considered here has a certain degree distribution and does not exhibit the nontrivial

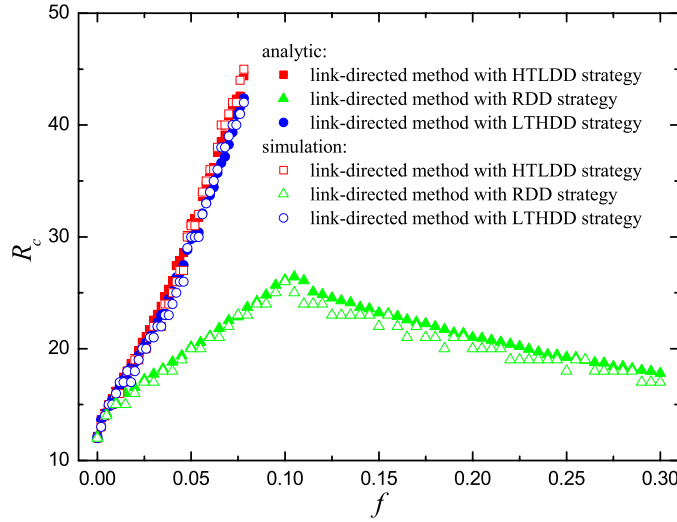


Figure 2. The critical generating rate R_c as a function f on the BA network for both the simulation and analysis. The network parameters and the number of run are the same as in figure 1.

degree–degree correlation. We expect to build up our experience for estimating the critical fraction f_c by performing extensive numerical simulations on the BA networks with different system’s size N and the parameter m . For the HTLDD and LTHDD strategies, we show the critical fraction f_c as the functions of both the system’s size N and the model parameter m in figures 3(a) and (b). With N increasing, f_c decreases which can be fitted in the following power law form:

$$f_c \propto N^\mu, \tag{5}$$

with $\mu = -0.375 \pm 0.008$; while f_c increases slowly when m increases, which obeys the scaling relation

$$f_c \propto \exp[\nu m], \tag{6}$$

with $\nu = 0.037 \pm 0.005$. Note that when f_c decreases with N increasing, one might think that the link-directed methods would be inefficient for huge-size networks. However, to our surprise, the link-directed method with the HTLDD or LTHDD strategy is still robust for huge-size networks, and the highest improved times of R_c (i.e. $R_c|_{f=f_c}/R_c|_{f=0.0}$) are independent of the network’s size. Although when f_c decreases with N increasing, the number of unidirectional links increases ($f_c \times mN \propto N^{(1+\nu)}$), which ensure that enough links connected the nodes with high load can be changed to unidirectional ones. The numerical evidence for this result is presented in figure 3(d), where we show the highest improved times as a function of N . For the RDD strategy, in general, f_c is larger than the optimal fraction f_o corresponding to the maximal network capacity. Thus, we focus on the optimal fraction f_o . From figure 3(c), we find that f_o decreases with N increasing, while increases with m increasing. Also the highest improved times (i.e. $R_c|_{f=f_o}/R_c|_{f=0.0}$) are independent of system sizes N (see figure 3(d)). Perhaps, these results would be useful to drop a hint for estimating the optimal fraction in the practical application of the link-directed method.

Next we study the change of the shortest path length affected by the link-directed method. As shown in figure 4, the average shortest path length L_{ave} under the global routing strategy increases after a fraction of links is limited to be unidirectional ones, and L_{ave} is a monotonically

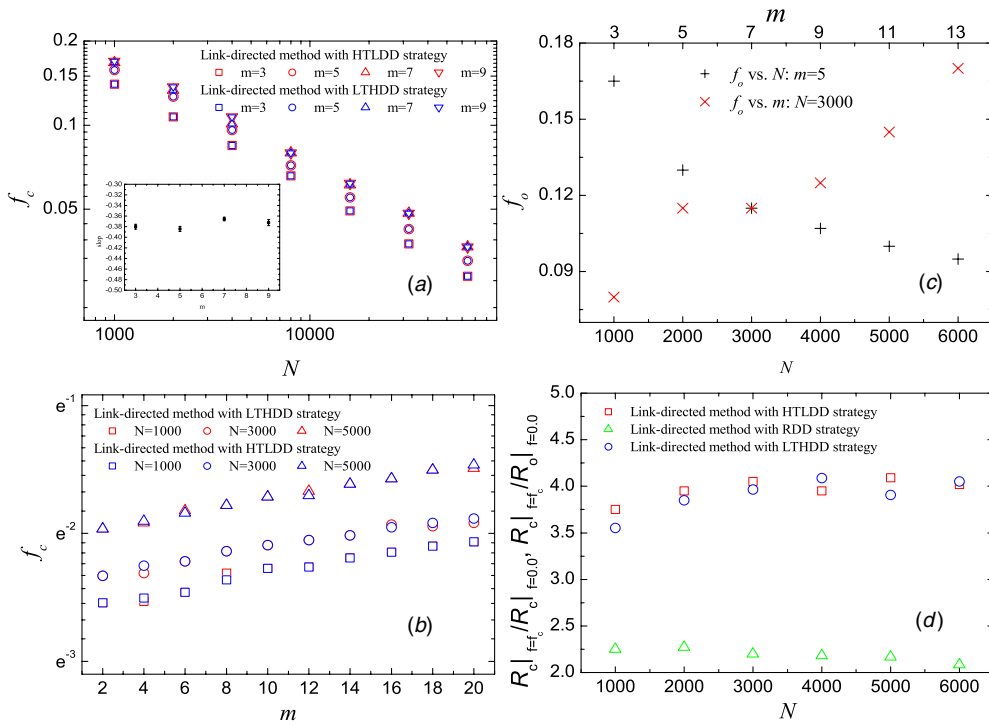


Figure 3. The average packet travel time $\langle T \rangle$ versus R for the link-directed method with three different DD strategies. The ratio of the number of directed links to the number of undirected links in the initial network $f = 0.07$. The network parameters and the number of runs are the same as in figure 1.

increasing function of f . It is not difficult to explain the increment of L_{ave} . Initially, most packets tend to pass through hub routers. After some key links become unidirectional ones, the packets have to change their path to other routers that are not so heavily linked, and thus L_{ave} increases. The link-directed method can enhance the overall traffic capacity, but at the cost of increasing the path length of the packets. However, although L_{ave} increases with f , the increments of L_{ave} are small and the small-world property $L_{ave} \sim \ln N$ [38] is still retained. The system capability in handling information is considerably enhanced at the cost of increasing the average path length. Such a sacrifice may be worthwhile when a system requires large R_c . In addition, when comparing the average path length of the packets to that of link-closing and link-deletion methods in [30, 31], one can note that the path length of the packets is littler than that for the link-closing method. Thus, the cost caused by the increment of the path length is lower than that for the link-closing method.

Figure 5 shows the variation of average packet travel time $\langle T \rangle$ after applying the link-directed method under global routing strategies. The results in figure 5(a) show that η and $\langle T \rangle$ indicate the same critical value R_c . When $R \leq R_c$, $\langle T \rangle$ increases slowly with R (see figure 5(b)), because the increment of the average path length making packets spend more time on the network is small. When $R > R_c$, $\langle T \rangle$ increases suddenly, which indicates that a traffic congestion occurs, and thus the packets have to spend a long waiting time in the process of transportation.

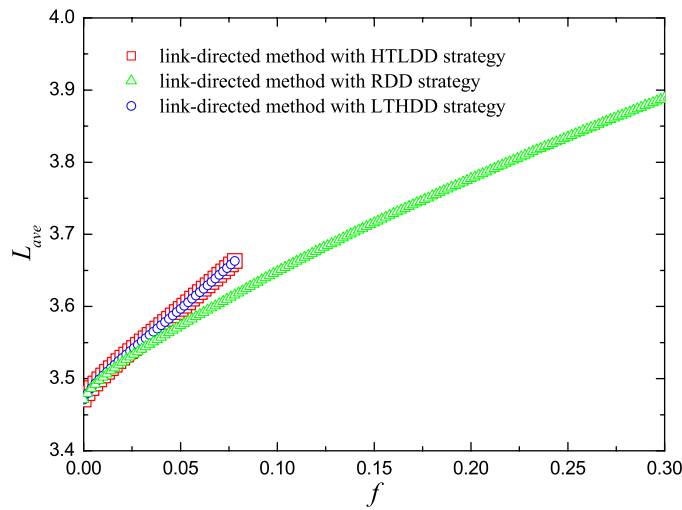


Figure 4. (a) System's size dependence of the critical fraction f_c for the link-directed methods with the HTLDD and LTHDD strategies. Inset plots the slopes of these curves. (b) Dependence of the critical fraction f_c on the model parameter m for the link-directed methods with the HTLDD and LTHDD strategies. (c) The optimal fraction f_o versus the system's size N and the model parameter m for the link-directed methods with the RDD strategy. (d) Size dependence of the highest improved times for the link-directed methods with the HTLDD, RDD and LTHDD strategies. The number of run are the same as in figure 1.

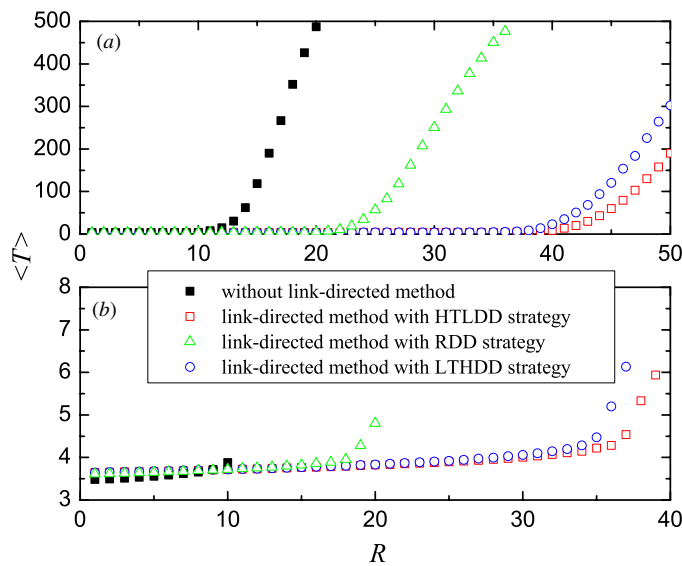


Figure 5. The average shortest path length L_{ave} versus f for the link-directed method with three different DD strategies. The network parameters and the number of runs are the same as in figure 1.

5. Conclusions and discussions

In conclusion, this paper introduces an alternative method to enhance the traffic capacity of scale-free networks by limiting some key links to the unidirectional ones at heavily loaded

times. This method can evidently enhance the overall traffic handling ability of scale-free networks under the condition of the global shortest-path routing strategy. Although the average path length L_{ave} and average travel time $\langle T \rangle$ of packets will increase with the number of unidirectional links, the method can help to achieve a system optimal traffic capacity in global routing. While our method is based on computer networks, we expect it to be relevant to other practical networks in general. For example, in communication and transportation networks the information or traffic flow is subject to temporal fluctuations [39, 40]. So the link-directed method can be applied to alleviate traffic congestion at times of high flux, and the links can be recovered to decrease the travel time of packets at times of low flux. We note that this method can be easily implemented in real communication systems since the link directing and recovering can be achieved by software. To make sure the packets will be forwarded accurately after directing the links, a new routing table can be broadcast before directing the links, so that each router may modify its routing table and each packet can be forwarded by its new shortest path in global routing.

Many studies focus on the traffic dynamics on the undirected networks which possesses the symmetrical interactions between any pair of interactive agents. Asymmetric interactions, however, widespread in natural and technological networks (e.g. the World Wide Web [41] and the E-mail network [42], etc), particularly when the network transports a flow or underlies collective behavior [43]. In addition, communication paths on various complex networks, even on the undirected networks, are not always symmetrical. In other words, communication path from agent i to agent j does not necessarily imply same communication path from agent j to agent i . To investigate both the traffic model and the routing strategy for the directed networks is thus significant in practice. The present work may be considered as only the first step to explore the traffic behaviors on the directed networks. A detailed inspection of the traffic dynamical behaviors on the directed networks will be done in the future works.

Finally, a remark should be made on the limitation of the application of our method. As has been mentioned above, the fraction of unidirectional links changed may have an upper limit since there is a possibility that the network is fragmentized into several disconnected components. However, due to complex and multiple properties for various real networks, it is difficult to estimate the upper limit or find an optimal value of the fraction to implement the present methods in practical. We expect further studies of these methods on some real networks to build up knowledge and experience.

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References

- [1] Albert R and Barabási A-L 2002 *Rev. Mod. Phys.* **74** 47–97
- [2] Dorogovtsev S N and Mendes J F F 2002 *Adv. Phys.* **51** 1079–187
- [3] Newman M E J 2003 *SIAM Rev.* **45** 167–256
- [4] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D-U 2006 *Phys. Rep.* **424** 175–308
- [5] Pastor-Satorras R and Vespignani A 2004 *Evolution and Structure of the Internet: A Statistical Physics Approach* (Cambridge: Cambridge University Press)
- [6] Watts D J and Strogatz S H 1998 *Nature* **393** 440–2
- [7] Amaral L A N, Scala A, Barthélémy M and Stanley H E 2000 *Proc. Natl Acad. Sci.* **97** 11149–52

- [8] Rosvall M, Trusina A, Minnhagen P and Sneppen K 2005 *Phys. Rev. Lett.* **94** 028701
- [9] Kalapala V, Sanwalani V, Clauset A and Moore C 2006 *Phys. Rev. E* **73** 026130
- [10] Guimerà R, Diaz-Guilera A, Vega-Redondo F, Cabrales A and Arenas A 2002 *Phys. Rev. Lett.* **89** 248701
- [11] Cholvi V, Laderas V, López L and Fernández A 2005 *Phys. Rev. E* **71** 035103
- [12] Singh B K and Gupte N 2005 *Phys. Rev. E* **71** 055103
- [13] Tadić B, Thurner S and Rodgers G J 2004 *Phys. Rev. E* **69** 036102
- [14] Tadić B and Thurner S 2004 *Physica A* **332** 566–84
- [15] Echenique P, Gómez-Gardeñes J and Moreno Y 2004 *Phys. Rev. E* **70** 056105
- [16] Wang W X, Wang B H, Yin C Y, Xie Y B and Zhou T 2006 *Phys. Rev. E* **73** 026111
- [17] Yan G, Zhou T, Hu B, Fu Z Q and Wang B H 2006 *Phys. Rev. E* **73** 046108
- [18] Chen Z Y and Wang X F 2006 *Phys. Rev. E* **73** 036107
- [19] de Moura A P S 2005 *Phys. Rev. E* **71** 066114
- [20] Eisler Z and Kertész J 2005 *Phys. Rev. E* **71** 057104
- [21] Goh K I, Kahng B and Kim D 2001 *Phys. Rev. Lett.* **87** 278701
- [22] Zhao L, Lai Y C, Park K and Ye N 2005 *Phys. Rev. E* **71** 026125
- [23] Hu M B, Wang W X, Jiang R, Wu Q S and Wu Y H 2007 *Phys. Rev. E* **75** 036102
- [24] Wang W X, Yin C Y, Yan G and Wang B H 2006 *Phys. Rev. E* **74** 016101
- [25] Yin C Y, Wang B H, Wang W X, Yan G and Yang H J 2006 *Eur. Phys. J. B* **49** 205–11
- [26] Toroczkai Z and Bassler K E 2004 *Nature* **428** 716
- [27] Arenas A, Diaz-Guilera A and Guimerà R 2001 *Phys. Rev. Lett.* **86** 3196
- [28] Tadić B, Rodgers G J and Thurner S 2007 *Int. J. Bifurcation Chaos* **17** 2363–85
- [29] Barabási A L and Albert R 1999 *Science* **286** 509–12
- [30] Zhang G Q, Wang D and Li G J 2007 *Phys. Rev. E* **76** 017101
- [31] Liu Z, Hu M B, Jiang R, Wang W X and Wu Q S 2007 *Phys. Rev. E* **76** 037101
- [32] Dorogovtsev S N 2001 *Nature* **410** 268–76
- [33] Wu Z X, Wang W X and Yeung K H 2008 *New J. Phys.* **10** 023025
- [34] Holme P, Kim B J, Yoon C N and Han S K 2002 *Phys. Rev. E* **65** 056109
- [35] Barrat A, Barthélémy M, Pastor-Satorras R and Vespignani A 2004 *Proc. Natl Acad. Sci.* **101** 3747–52
- [36] Macdonald P J, Almaas E and Barabási A L 2005 *Europhys. Lett.* **72** 308–14
- [37] Freeman L C 1979 *Soc. Netw.* **1** 215–39
- [38] Chung F and Lu L 2001 *Adv. Appl. Math.* **26** 257–79
- [39] de Menezes M A and Barabási A L 2004 *Phys. Rev. Lett.* **92** 028701
- [40] Heide D, Schäfer M and Greiner M 2008 *Phys. Rev. E* **77** 056103
- [41] Liu J G, Dang Y Z, Wang Z T and Zhou T 2005 arXiv:physics/0510064
- [42] Newman M E J, Forrest S and Balthrop J 2002 *Phys. Rev. E* **66** 035101
- [43] Klemm K and Bornholdt S 2005 *Proc. Natl Acad. Sci.* **102** 18414–9